ANNALISA CONVERSANO, Maximal definably compact subgroups of definable groups in o-minimal structures.

Department of Mathematics and Computer Science, University of Siena, Piano dei Mantellini 44, Italy.

E-mail: conversa@mail.dm.unipi.it.

Definable groups in o-minimal structures have been studied by many authors, especially in the definably compact case. We prove that if G is a definably connected definable group in an o-minimal expansion of a real closed field M, then the following are equivalent:

- 1. G has maximal definably compact definable subgroups.
- 2. G = KH, with K definably compact and H torsion-free definable subgroups.

3. Every 0-subgroup of G is definably compact.

We show that in any case G has a unique maximal normal torsion-free definable subgroup $N \triangleleft G$ and G/N = KH, with K definably compact and H torsion-free definable subgroups. Therefore G is definably homeomorphic to $K \times M^s$, where $s = \dim N + \dim H$, and G is definably homotopically equivalent to K.

Moreover, for every maximal 0-subgroup A of G there is a definable subgroup P < G which is a (maximal) union of conjugates of A and such that G = PH, for some maximal torsion-free definable subgroup H of G. This shows how maximal 0-subgroups play the same role in definable groups as maximal tori do in real Lie groups, even though a definable 0-group might be not definably compact.