► ADAM KAY, Incompletability of formal linguistics.

Department of Mathematics and Statistics, Open University, Milton Keynes, MK7 6BJ, United Kingdom.

E-mail: afokay@gmail.com.

With a simple Gödel-like coding of the vocabulary of a natural language into sets, formal linguistics can be entirely expressed within the theory of hereditarily finite sets (HF). The main definition assuring that linguistic properties are represented is of the triconditional $T \iff Q \iff \Phi$, between tree structures T, grammatical categories Q, and the formulas Φ representing the property of inclusion in some category. The triconditional assures that grammatical categories are associated with labeled finite rooted trees and logical formulas. The concept grammatical is then a decidable relation between expressions of a language (sets), and their structural descriptions (sets).

When the formulas Φ are added to HF as axioms the result is a grammar G. Grammatical sentences are thus analogous to true sentences in a formal logic, as usual. After giving a variety of grammar formalisms as examples and showing how the standard model \mathfrak{S} of HF adapts to each grammar formalism, we show that to develop the concept of *ungrammatical* in G requires meta-grammatical categories M, defined in terms of Q. A simple diagonalization argument of Tarski's then immediately shows G (equipped with M) to be incomplete.

We conclude that formal linguistics, plus the concept *ungrammatical*, is incomplete, that the attempt to determine the formal rules underlying use of natural language is incompletable, and that the traditional goal of descriptive adequacy must be rethought.