

- MARGARITA LEONTYEVA, *Decidable Boolean algebras of elementary characteristics (1,0,1)*.

Department of Mechanics and Mathematics, Novosibirsk State University, Russian Federation.

E-mail: miss-leontieva@ngs.ru.

A computable Boolean algebra is said to be *n-constructive* if there exists an algorithm determining whether the given tuple satisfies given finite Σ_n -formula. A *strongly constructive* Boolean algebra is one for which such an algorithm exists for all formulas of the predicate calculus. Boolean algebra is *decidable* if there exists its strongly constructive isomorphic copy.

If \mathfrak{A} is a Boolean algebra, then $\text{At}(\mathfrak{A})$ is the set of atoms of \mathfrak{A} , $\text{Atm}(\mathfrak{A})$ is the ideal of atomic elements, $\text{Als}(\mathfrak{A})$ is the ideal of atomless elements, $\text{E}(\mathfrak{A}) = \text{Atm}(\mathfrak{A}) + \text{Als}(\mathfrak{A})$ is the Ershov-Tarski ideal and F is Frechet ideal.

Relations between computability of $\text{At}(\mathfrak{A})$, $\text{Atm}(\mathfrak{A})$, $\text{Als}(\mathfrak{A})$, $\text{E}(\mathfrak{A})$ and decidability of Boolean algebra \mathfrak{A} began to be studied in the articles of Ershov in 1964.

Let \mathfrak{A} be a computable Boolean algebra of elementary characteristics (1,0,1). This means that \mathfrak{A}/E is a nontrivial atomless Boolean algebra. Let S be a subset of the set $\{\text{At}, \text{Atm}, \text{Als}, \text{E}\}$. We consider a general question: if the predicates in S are computable in \mathfrak{A} then can we state that \mathfrak{A} is decidable? It was proved earlier that if $S = \{\text{At}, \text{Als}\}$ then the answer is yes, and if $S = \{\text{At}\}$ or $S = \{\text{Als}, \text{Atm}, \text{E}\}$ then the answer is no. In theorems 1 and 2 we complete this theme.

Theorem 1 Let \mathfrak{A} be a computable Boolean algebra of elementary characteristics (1, 0, 1). If $\text{At}(\mathfrak{A})$ and $\text{Atm}(\mathfrak{A})$ are computable then \mathfrak{A} is decidable.

Theorem 2 There exists a computable Boolean algebra \mathfrak{A} of elementary characteristics (1, 0, 1) with computable $\text{At}(\mathfrak{A})$ and $\text{E}(\mathfrak{A})$ which is not decidable.

During the work with theorem 2 the following description of Δ_6^0 -computable Boolean algebras was obtained. Let $T = (\text{Atm} \rightarrow \text{F}) + \text{Atm}$, where $\text{Atm} \rightarrow \text{F} = \{x | \forall z \leq x (z \in \text{F} \vee z \notin \text{Atm})\}$.

Theorem 3 \mathfrak{A} is Δ_6^0 -computable Boolean algebra if and only if there exists a computable Boolean algebra \mathfrak{C} such that $\mathfrak{C}/T \cong \mathfrak{A}$.