▶ STEFAN HETZL, *On the form of witness terms.*
INRIA Saclay – Île-de-France, École Polytechnique – LIX, 91128 Palaiseau, France.
E-mail: hetzl@lix.polytechnique.fr.

Cut-elimination is an inherently non-deterministic process. At each stage one has the choice between different cuts to reduce and – for a single cut – there are different ways of reducing it. This formal non-determinism has the effect that depending on the chosen cut-elimination procedure, mathematically different normal forms may be produced. Understanding the possible span of the results of cut-elimination procedures is therefore of fundamental importance for judging proof analyses based on these methods.

The mathematical content of a cut-free proof is fully contained in the information how to instantiate which quantifiers, i.e. in the witness terms. They therefore provide adequate means for relating different cut-free proofs.

We analyze the development of terms during cut-elimination in first-order logic and Peano arithmetic. The main result is a characterization of the form of witness terms in cut-free proofs in terms of structured combinations of basic substitutions that are read off from the proof with cuts.

Based on this result, it is shown that each proof with cuts induces a regular tree grammar s.t. every witness term computable by cut-elimination can also be computed by the grammar. As a second application of this result, a class of proofs in first-order logic is shown to have only elementary cut-elimination (while cut-elimination in the worst case is non-elementary). From the algorithmic point of view, we obtain a method for computing witness terms that circumvents cut-elimination and has several advantages, it allows for example to find the shortest witness term. All of the above results also apply to proofs of Σ_1^0-formulas in Peano arithmetic.