▸ ULRICH KOHLENBACH, *Applied Proof Theory: Proof Interpretations and Their Use in Mathematics*.
Department of Mathematics
Technische Universität Darmstadt
Schlossgartenstraße 7
D-64289 Darmstadt, Germany.
E-mail: kohlenbach@mathematik.tu-darmstadt.de.

This tutorial gives an introduction to an applied form of proof theory that has its roots in G. Kreisel's pioneering ideas on 'unwinding proofs' but has evolved into a systematic activity only during that last 10 years. The general approach is to apply proof theoretic techniques (notably specially designed proof interpretations) to concrete mathematical proofs with the aim of extracting new quantitative results (such as effective bounds) as well as new qualitative uniformity results from (prima facie ineffective) proofs. During the last years logical metatheorems have been developed which guarantee the extractability of highly uniform bounds from large classes of proofs in nonlinear analysis. 'Uniform' here refers to the fact that the bounds are independent from parameters in abstract classes of spaces as long as some local bounds on certain metric distances are given. The classes of structures covered include metric, hyperbolic (in the sense of Kirk and Reich), δ-hyperbolic (in the sense of Gromov), CAT(0), normed, uniformly convex spaces as well $\mathbb{R}$-trees (in the sense of Tits). To achieve uniformity in the absence of compactness it is crucial to exploit the fact that the proofs in question do not make use of any separability assumptions. Applications to various parts of mathematics have led to new results in approximation theory, nonlinear analysis, metric fixed point theory, geodesic geometry, ergodic theory and topological dynamics.

Part I outlines the general program of this kind of applied proof theory motivated by examples from different parts of mathematics. We will emphasize the close relation between this program and T. Tao's concept of 'finitary' analysis.

Part II develops some of the main logical metatheorems that guarantee the extractability of uniform bounds from large classes of proofs.

Part III surveys some of the most interesting applications to nonlinear analysis, ergodic theory and topological dynamics.

The tutorial does not require specific proof-theoretic prerequisites.

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