▶ ITAY NEEMAN, Forcing with ultrafilters.

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In 1989 Woodin and others asked whether, for a singular cardinal κ of cofinality ω , the tree property at κ^+ implies the singular cardinal hypothesis at κ . At the time, the only models in which SCH failed were obtained by singularizing a regular cardinal where the GCH fails, and the question was intended to test whether this was the only way. Later results by Gitik–Magidor showed that there are other ways, yet the test question itself persisted. It has since become a motivator for several results on square principles, considered possible intermediaries on the way from the tree property to the SCH.

We settle the question in this talk. We show that the tree property at κ^+ does not imply SCH at κ . The model where the tree property holds and SCH fails is obtained by *forcing with ultrafilters*. We give a survey of this technique and of related results, starting with some of the most well known instances. The talk is self contained and accessible to a general audience in mathematical logic.