▸ ANDRÉ NIES, *Applying randomness to computability.*
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We study the complexity and randomness aspects of sets of natural numbers. Traditionally, computability theory is concerned with the complexity aspect. However, computability-theoretic tools can also be used to introduce mathematical definitions of randomness of a set; further, once defined, these notions can be studied by considering their interplay with the complexity aspect of a set.

There is also an interaction in the converse direction: concepts and methods from randomness enrich computability theory.

The first tutorial treats the interaction from computability to randomness. The second and third tutorials cover the converse interaction, which is in the focus of recent research. Most of the results can be found in [5].

A lowness property of a set B specifies a sense in which B is close to being computable. One of the most striking examples of applying randomness in computability is the discovery of lowness properties defined in a random-theoretic way. Surprisingly, classes with very different definitions were shown to coincide. The first property defined in this way was lowness for Martin-Löf randomness: each ML-random set is already ML-random relative to B [4]. This property was shown to be equivalent to various other lowness properties, such as a base for ML-randomness, and being low for weak 2-randomness. In a different vein, it is also equivalent to K-triviality [6], a property that expresses being far from random. Before these coincidences were proven, each of the classes was studied separately. In particular, researchers showed the existence of a promptly simple set in the class. The cost function method arose to give a general framework for these constructions.

Recent research centers on subclasses of the K-trivials. The following is a purely computability-theoretic lowness property: B is strongly jump traceable if there is a c.e. set of possible values for $J^B(x)$ that is finite of small size [2]. Nonetheless, in [1] it was proved that the c.e. strongly jump traceable sets form a proper subideal of the c.e. K-trivials. Recent work [3] suggests that the sets in this class are close to being computable because, in some sense, many ML-random oracles compute them. In [3] we prove a number of coincidences of strong jump traceability (for c.e. sets) with properties that formalize being computed by many ML-random sets.

We also discuss the possibility of natural ideals properly in between the strongly jump-traceable and the K-trivial c.e. sets. We look at reducibilities weaker than Turing that induce the lowness properties above. For instance $B \leq_{LR} C$ means that every C-random set is also B-random [6]. The least LR degree consists of the sets that are low for ML-randomness. In both areas open questions abound.

Finally, we will take a look at analogs of the foregoing results for "higher" randomness notions defined in terms of effective descriptive set theory.

There will be handouts and informal exercise sessions to make the material accessible to students.

[1] Cholak, P., Downey, R., and Greenberg, N., *Strongly jump-traceability I: the computably enumerable case*, ***Advances in Mathematics***, vol. 217 (2008), 2045–2074.

[2] Figueira, S., Nies, A., and Stephan, F. , *Lowness properties and approximations of the jump*, ***Ann. Pure Appl. Logic***, vol. 152 (2008), 251–66.

[3] Greenberg, N., Hirschfeldt, D., and Nies, A., *Characterizing strong jump traceability via randomness*, to appear.

[4] Kučera, A. and Terwijn, S., *Lowness for the class of random sets*, ***J. Symbolic Logic***, vol. 64 (1999), 1396–1402.

[5] Nies, A., *Computability and randomness*, Oxford Logic Guides, Oxford University Press, 2009, 444 pages.

[6] Nies, A., *Lowness properties and randomness*, ***Advances in Mathematics***, vol. 197 (2005), 274–305.