ASSAF RINOT, Diamond, non-saturation, and weak square principles.
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We report on results from [1] and [2] concerning the effect of weak square principles to guessing principles. Let $\operatorname{Refl}_{\lambda}$ denote the assertion that every stationary subset of $\{\alpha < \lambda^+ \mid \operatorname{cf}(\alpha) = \operatorname{cf}(\lambda)\}$ reflects. A corollary to the results that we shall discuss in our talk is the following.

Theorem. For a singular cardinal λ :

- 1. GCH + Refl_{λ} + $\Box_{\lambda}^* \Rightarrow \diamondsuit_{\lambda^+}^*$;
- 2. $\operatorname{GCH} + \operatorname{Refl}_{\lambda} + \operatorname{SAP}_{\lambda} \not\Rightarrow \diamondsuit_{\lambda^+}^*;$
- 3. GCH + Refl_{λ} + SAP_{λ} \Rightarrow \diamond ⁵_S for every stationary S \subseteq λ^+ ;
- 4. GCH + Refl_{λ} + AP_{λ} \Rightarrow \diamondsuit_S for every stationary $S \subseteq \lambda^+$.

In addition, we prove that SAP_{λ} (and hence \Box_{λ}^{*}) implies that $\text{NS}_{\lambda^{+}} \upharpoonright S$ is nonsaturated for every $S \subseteq \lambda^{+}$ that reflects stationarily often. We prove that the failure of a guessing principle introduced by Džamonja and Shelah is equivalent to the failure of Shelah's strong hypothesis. We also provide two (negative) answers to a question of König, Larson and Yoshinobu; one in the presence of GCH, and one in its absence.

[1] M. GITIK AND A. RINOT, The failure of diamond on a reflecting stationary set, *preprint*, 2009.

[2] A. RINOT, A relative of the approachability ideal, diamond and non-saturation, *preprint*, 2009.