RICHARD A. SHORE, The Gödel Lecture, Logic Colloquium '09 Reverse Mathematics: the Playground of Logic.

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The enterprise of calibrating the strength of theorems of classical mathematics, primarily in terms of the (set existence) axioms needed to prove them, was begun by Harvey Friedman in the 1970's. It is now called Reverse Mathematics as, to prove that some set of axioms is actually necessary to establish a given theorem, one reverses the standard paradigm by proving that the axioms follow from the theorem (in some weak base theory). The original motivations for the subject were foundational and philosophical. It has become a remarkably fruitful and successful endeavor supplying a framework for both the philosophical questions about existence assumptions and foundational or mathematical ones about construction techniques needed to construct objects that the theorems assert exist.

There is one common base theory and four standard systems over it. Most theorems of classical mathematics have turned out to be equivalent to one of these systems. We will briefly describe this state of affairs and an alternative view of the calibration system based on computability theoretic measures. We will also see that more recent work has provided examples that fall outside the standard systems at both the bottom and top. The examples are drawn from combinatorics and various areas of logic. The techniques employed come from all the branches of mathematical logic: proof theory, model theory, set theory and recursion theory.