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Dependence is a common phenomenon, wherever one looks: ecological systems, astronomy, human history, stock markets—but what is the logic of dependence? In this talk I will outline a systematic logical study of dependence, starting from the basic concept of functional dependence in database theory. Dependence logic does not obey the Law of Excluded Middle, as its semantic power is the same as that of existential second order logic. In fact, there is a somewhat unexpected connection to intuitionistic logic (joint work with S. Abramsky). Even very basic model theoretic properties of dependence logic on infinite domains depend on large cardinals and deep set-theoretical facts. On the other hand, on finite domains dependence logic provides an alternative language for non-deterministic polynomial time queries.

The basic new ingredient of dependence logic over and above first order logic is the new form of atomic formulas called dependence atoms. An example is $=(x, y, z)$ with the meaning "z depends on and only on x and y". Note that we can think of x,y and z as features of individuals, as in the sentence "The salary (z) is determined by the rank (x) and number of years of employment (y)".

One of the guiding principles is that dependence does not manifest itself in a single event or observation. Instead, we use semantics, due to W. Hodges ([3]), where the basic concept is a set (or "team") of observations. Examples of such sets are a set of chess games between Susan and Max, a set of records of stock exchange transactions of a particular dealer, a set of possible histories of mankind written as decisions and consequences, etc. As these examples show, the basic intuition is a two-person game. In a game a play is built up from the choices of the players. By looking at many plays we can learn about the players and discern what kind of dependences the moves of the players exhibit. The semantics of disjunction is characteristic: a set satisfies $\phi \vee \psi$ if the sets splits into two parts, one of which satisfies ϕ and the other satisfies ψ. The splitting reflects a player choosing in some plays ϕ and in some ψ.

Dependence logic is just one example of adding the concept of dependence to logic. This opens up possibilities to apply logic to situations, such as Arrow's Paradox of social choice, where interaction rather than plain truth is at stake.

[1] L. Henkin. Some remarks on infinitely long formulas. In *Infinitistic Methods (Proc. Sympos. Foundations of Math., Warsaw, 1959)*, pages 167–183. Pergamon, Oxford, 1961.

[2] Jaakko Hintikka. *The principles of mathematics revisited.* Cambridge University Press, Cambridge, 1996. Appendix by Gabriel Sandu.

[3] Wilfrid Hodges. Compositional semantics for a language of imperfect information. *Log. J. IGPL*, 5(4):539–563 (electronic), 1997.

[4] Jouko Väänänen. *Dependence logic*, volume 70 of *London Mathematical Society Student Texts.* Cambridge University Press, Cambridge, 2007.