Definability in the Real Universe

S. Barry Cooper - *Logic Colloquium 2009*, Sofia, Bulgaria, Wednesday, August 5, 2009

Definability in the Real Universe -

Hans Reichenbach
(1891-1953)

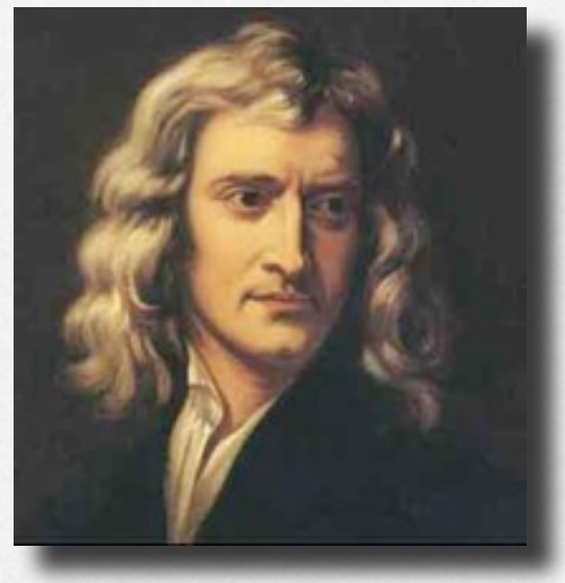
- How do scientists represent and establish control over information about the Universe

- How does the Universe itself exercise control over its own development ... or more feasible:

- How can we reflect that control via our scientific and mathematical representations

Computability versus descriptions

- Newton onwards – overarching aim of science became the extraction of the computable content of the world … theories which computably predict, capturing truth via proofs …



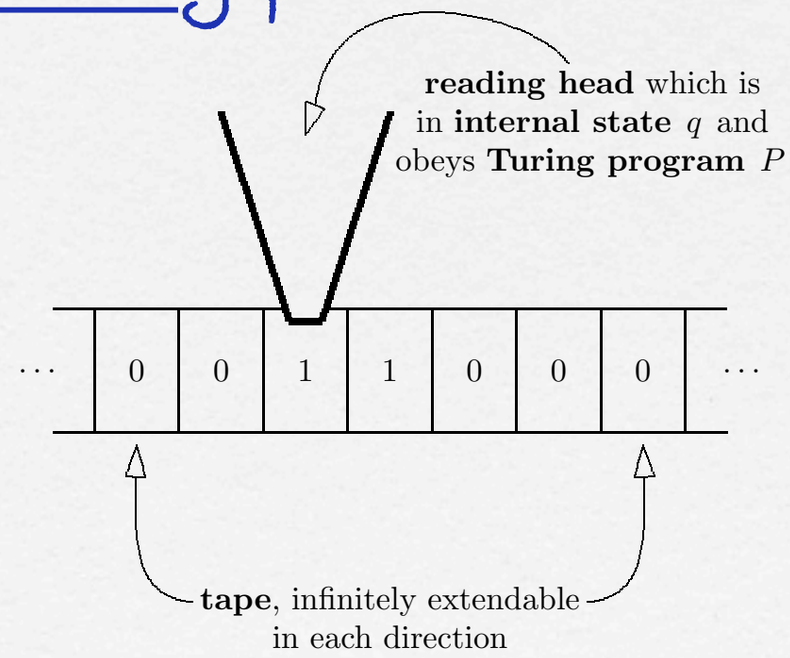
- Einstein [p.54, `Out of My Later Years', 1950]: "When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them."

Turing's model and Incomputability

- 1936 - Turing's *machines* arrive

- Points to a model of *computational* natural processes within *structures which are countably presented*

But - techniques for presenting machines give the *Universal Turing machine* - and *incomputable objects*



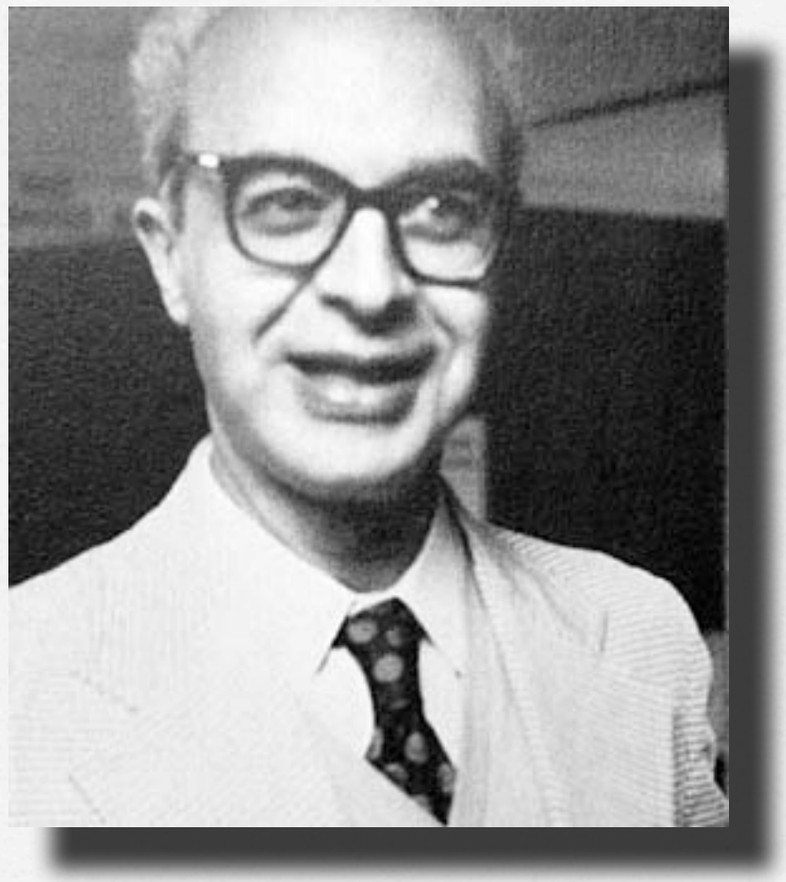
The Real Universe as Discipline Problem

The great success of modern computers as all-purpose algorithm-executing engines embodying Turing's universal computer in physical form, makes it extremely plausible that the abstract theory of computability gives the correct answer to the question 'What is a computation?', and, by itself, makes the existence of any more general form of computation extremely doubtful.

Martin Davis [2004], The myth of hypercomputation. In Alan Turing: Life and legacy of a great thinker (C. Teuscher, ed.), Springer-Verlag

A dissenting voice …

1970 - Georg Kreisel proposes a collision problem related to the 3-body problem, which might result in *"an analog computation of a non-recursive function"*



A dissenting voice ...

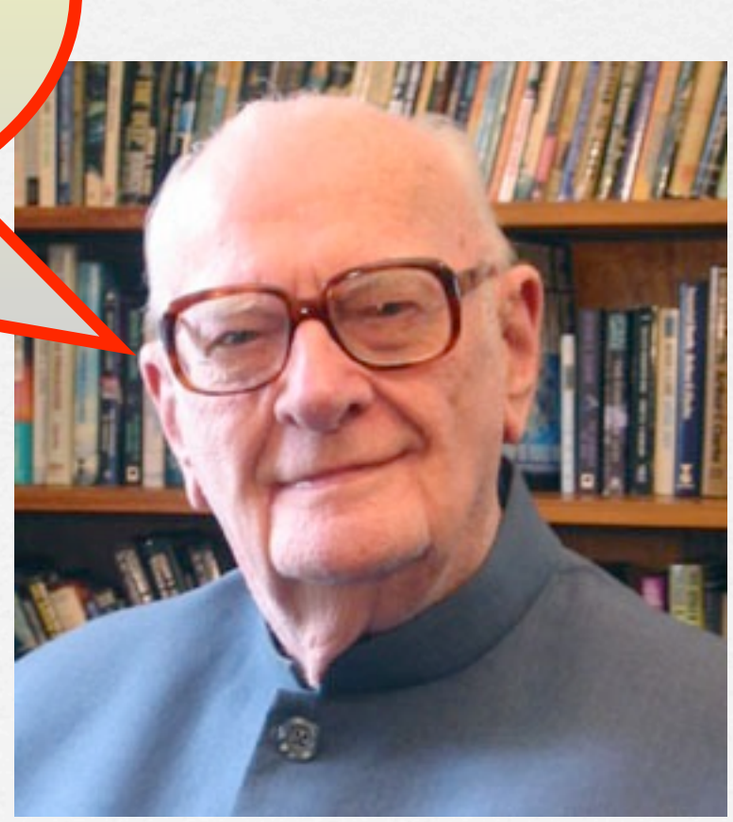
- Persistence of problems of predictability - quantum uncertainty, emergent phenomena, chaos and strange attractors, relativity and singularities (black holes)

- Renewed interest in analog and hybrid computing machines leading to: "... the classical Turing paradigm may no longer be fully appropriate to capture all features of present-day computing."

- J. van Leeuwen, J. Wiedermann, The Turing Machine Paradigm in Contemporary Computing. In Mathematics Unlimited - 2001 and Beyond, LNCS, 2000

Clarke's First Law:

When a distinguished but elderly scientist states that something is possible, he is almost certainly right. When he states that something is impossible, he is very probably wrong.

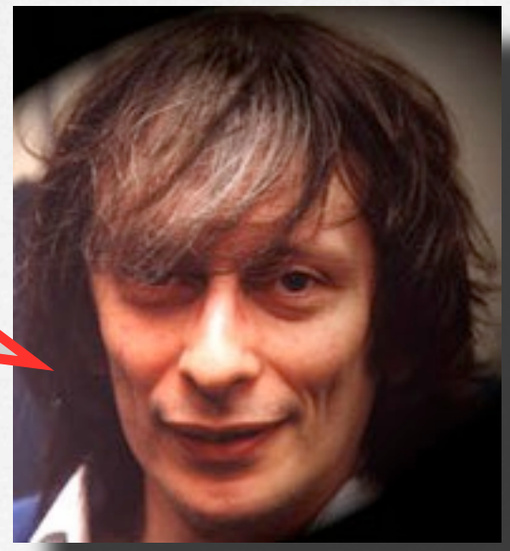


Arthur C. Clarke [1962], Hazards of Prophecy. The Failure of Imagination. In "Profiles of the Future", Gollancz, London, 1962

1. The Challenge of Quantum Theory ...

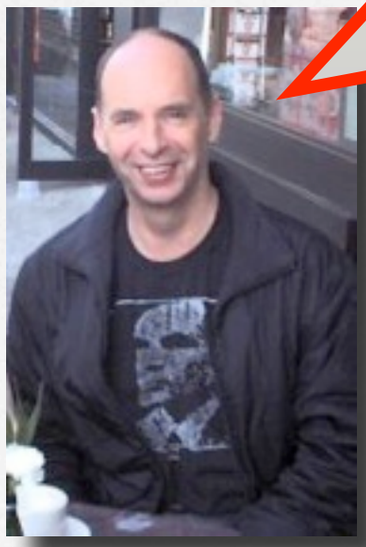
- Successful reduction of "natural" examples to the Turing model - e.g. quantum computation (David Deutsch, 1985)

I am sure we will have [conscious computers], I expect they will be purely classical, and I expect that it will be a long time in the future. Significant advances in our philosophical understanding of what consciousness is, will be needed.



Question and Answers with David Deutsch, on New.Scientist.com News Service, December, 2006

"Von Neumann's axioms distinguished the **U** (unitary evolution) and **R** (reduction) rules of quantum mechanics. Now, quantum computing so far (in the work of Feynman, Deutsch, Shor, etc) is based on the **U** process and so computable. It has not made serious use of the **R** process: the unpredictable element that comes in with reduction, measurement, or collapse of the wave function."



Andrew Hodges

in "What would Alan Turing have done after 1954?", from Teuscher,

"Alan Turing: Life and legacy of a great thinker"

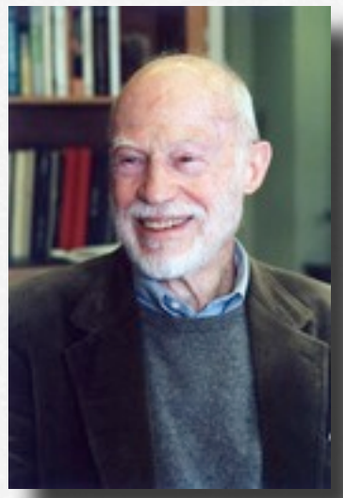
Schrödinger's disappearing worlds

Processes for change of wave equation describing quantum state of a physical system:

- Deterministic continuous evolution via Schrödinger's equation - involves superpositions of basis states
- Probabilistic non-local discontinuous change due to measurement - observe a jump to a single basis state

Interpretation?? (where do the other states go)

Many Worlds Interpretation

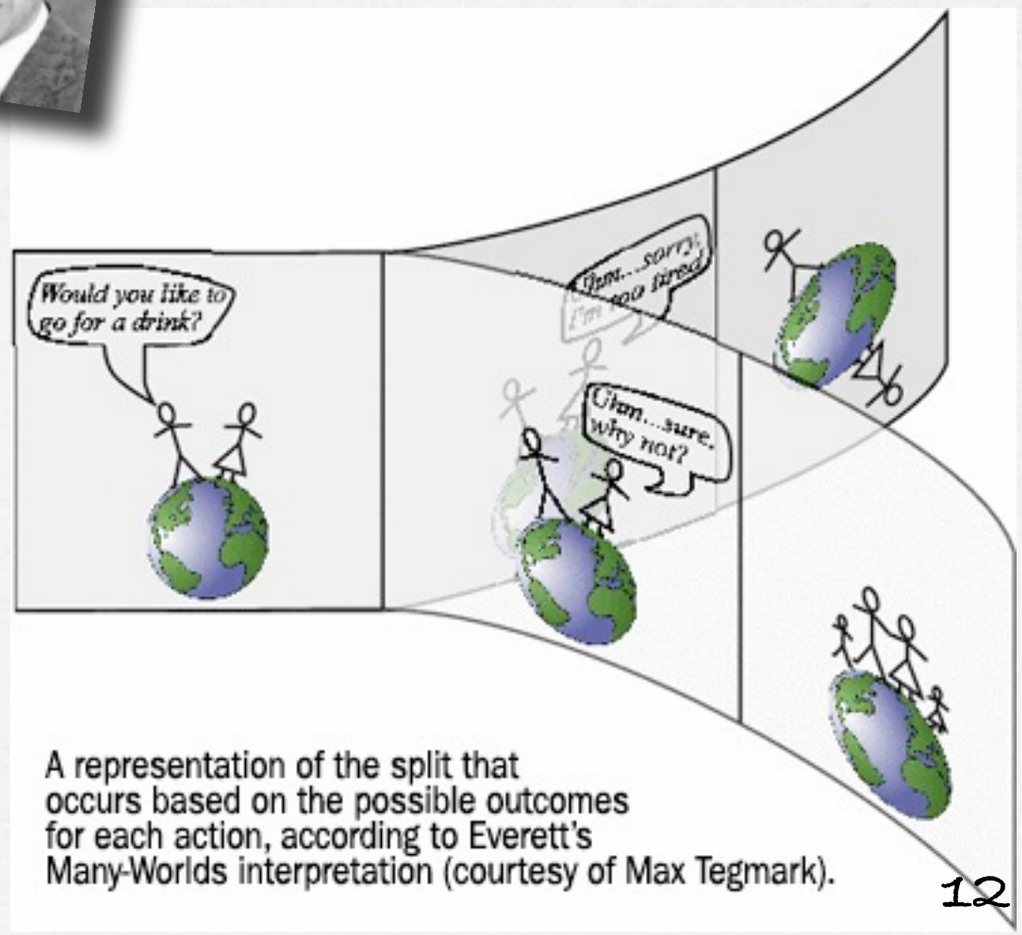


Bryce DeWitt

John Wheeler

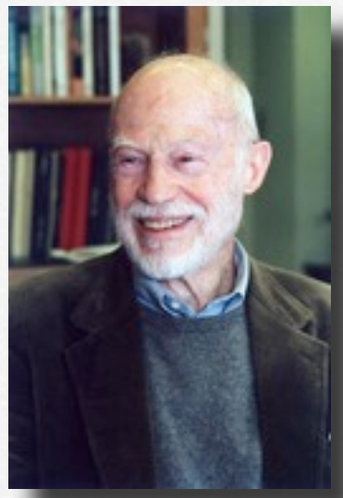
Hugh Everett III

(Nov.11, 1930- July 19, 1982)



A representation of the split that occurs based on the possible outcomes for each action, according to Everett's Many-Worlds interpretation (courtesy of Max Tegmark).

Many Worlds Interpretation



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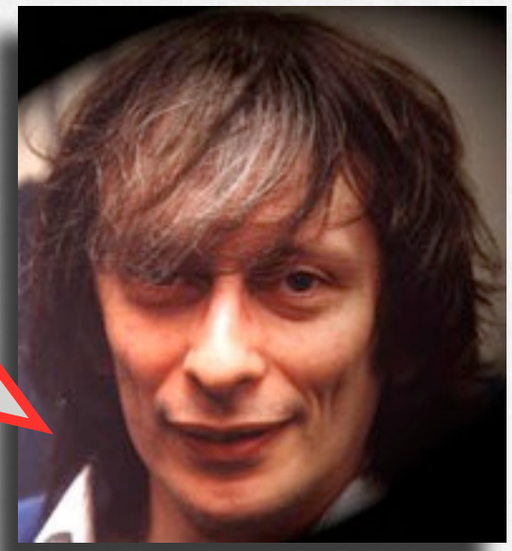
Mark Oliver Everett -
lead singer/guitarist of EELS

The Multiverse + Anthropic Principle

... understanding the multiverse is a precondition for understanding reality as best we can. Nor is this said in a spirit of grim determination to seek the truth no matter how unpalatable it may be ... It is, on the contrary, because the resulting world-view is so much more integrated, and makes more sense in so many ways, than any previous world-view, and certainly more than the cynical pragmatism which too often nowadays serves as surrogate for a world-view amongst scientists.



David Deutsch, The Fabric of Reality, Allen Lane, 1997, p.48

The Multiverse

The issue of what is to be regarded as an ensemble of 'all possible' universes is unclear, it can be manipulated to produce any result you want ... The argument that this infinite ensemble actually exists can be claimed to have a certain explanatory economy (Tegmark 1993), although others would claim that Occam's razor has been completely abandoned in favour of a profligate excess of existential multiplicity, extravagantly hypothesized in order to explain the one universe that we do know exists.

George Ellis, The Unique Nature of Cosmology, in Revisiting the Foundations of Relativistic Physics (eds. Abhay Ashtekar et al), Kluwer, 1996, p.198

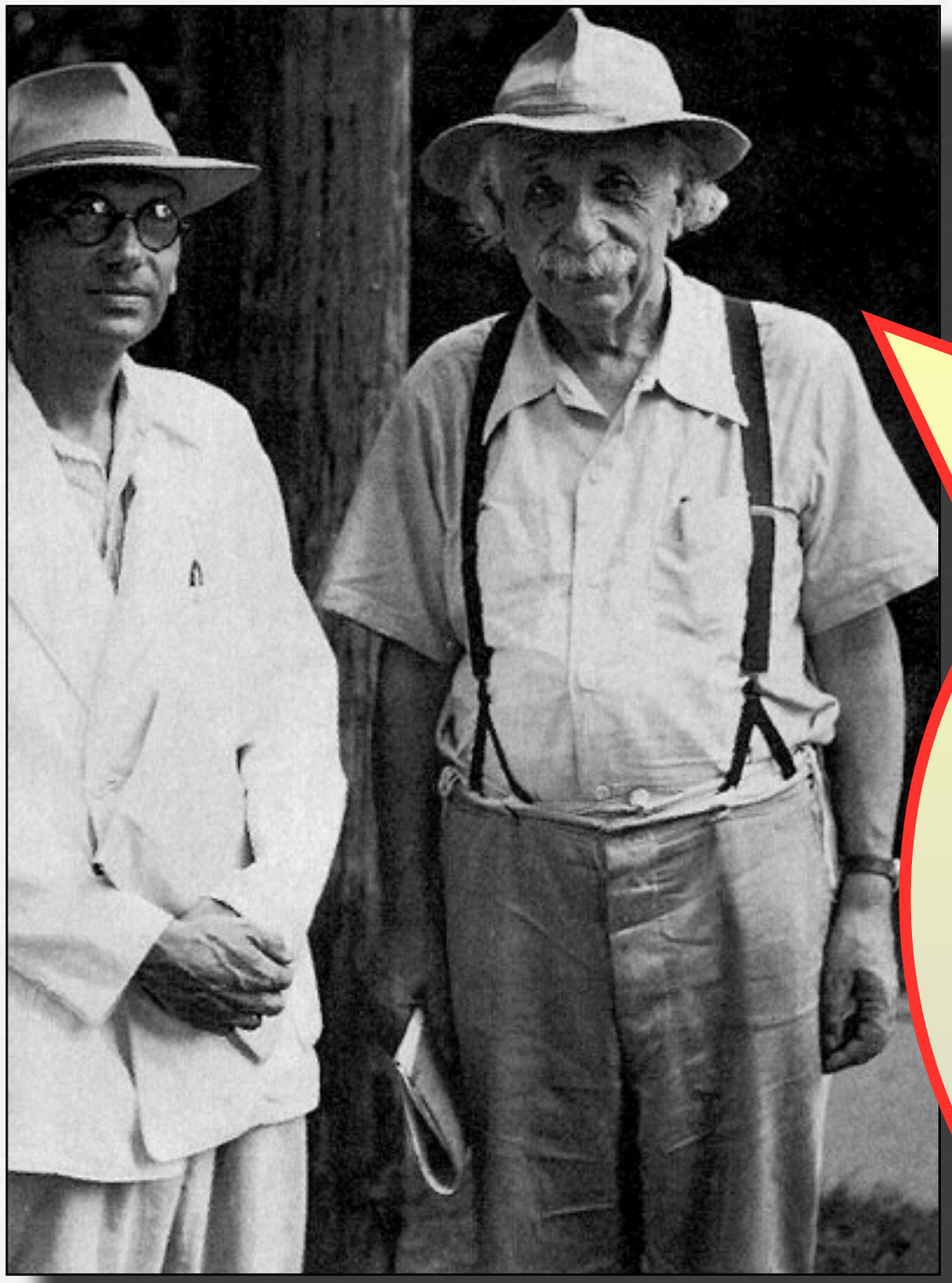
Back In One World ...

If the creation of the universe can be described as a quantum process, we would be left with one deep mystery of existence: What is it that determined the laws of physics?

A.H. Guth, *The Inflationary Universe - The Quest for a New Theory of Cosmic Origins*, Addison-Wesley, 1997

One way of thinking about what is unsatisfactory about the standard model is that it leaves seventeen non-trivial numbers still to be explained, ...

Peter Woit: *Not Even Wrong - The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics*, Jonathan Cape, 2006



From A. Einstein: "Autobiographical Notes", in "Albert Einstein: Philosopher-Scientist" (P. Schilpp, ed.), Open Court Publishing, 1969, p.63

... I would like to state a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e. intelligibility, of nature ... nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory) ...

Strong Determinism

[According to Strong Determinism] ... all the complication, variety and apparent randomness that we see all about us, as well as the precise physical laws, are all exact and unambiguous consequences of one single coherent mathematical structure.

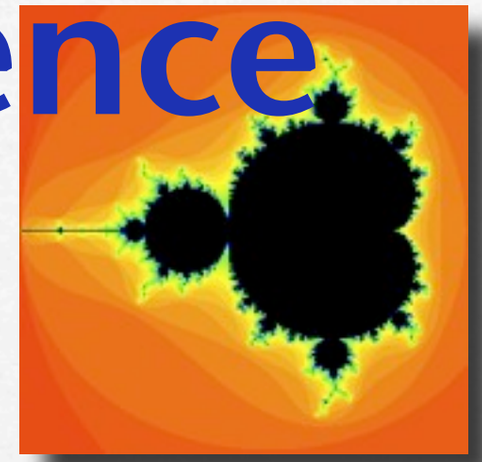
Roger Penrose: *Quantum physics and conscious thought*, in *Quantum Implications: Essays in honour of David Bohm* (B.J. Hiley and F.D. Peat, eds.), pp.106-107

Strong Determinism

For computability theoretic ramifications:

C. Calude, D.I. Campbell, K.Svozil, D. Stefanescu:
Strong determinism vs. computability,
in W. Depauli-Schimanovich, E. Koehler, F. Stadler (eds.),
The Foundational Debate, Complexity and Constructivity
in Mathematics and Physics,
Kluwer, Dordrecht, 1995, 115-131.

2. The Emergence Challenge ...

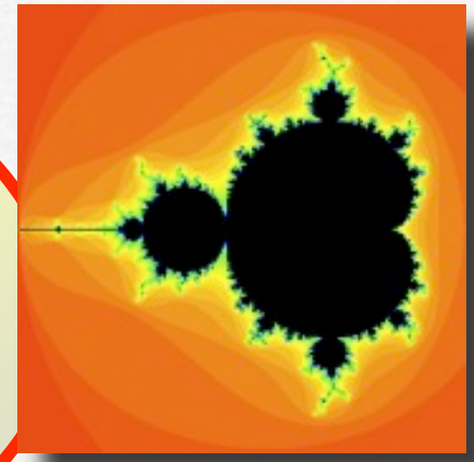


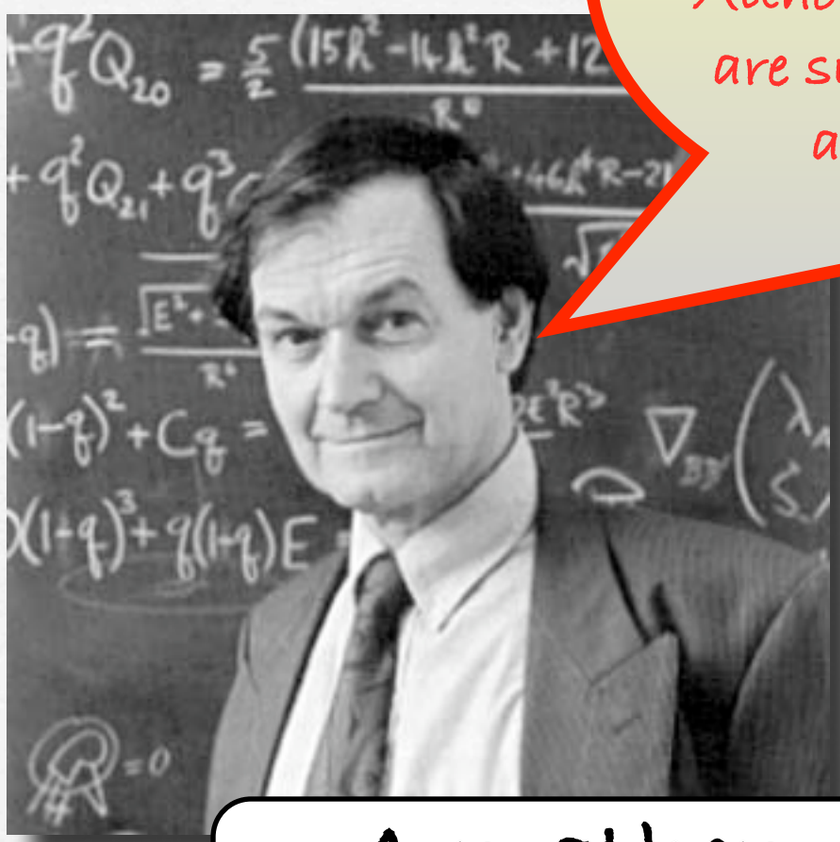
- Growth of Chaos theory, generation of informational complexity via very simple rules, accompanied by the emergence of new regularities - e.g. Robert Shaw's dripping tap[1984]

- Link between structures in nature, and mathematical objects, such as the Mandelbrot and Julia sets

- Penrose, Smale - computability of Mandelbrot, Julia sets?

Now we witnessed ...
a certain extraordinarily
complicated looking set, namely the
Mandelbrot set.
Although the rules which provide its definition
are surprisingly simple, the set itself exhibits
an endless variety of highly elaborate
structures.

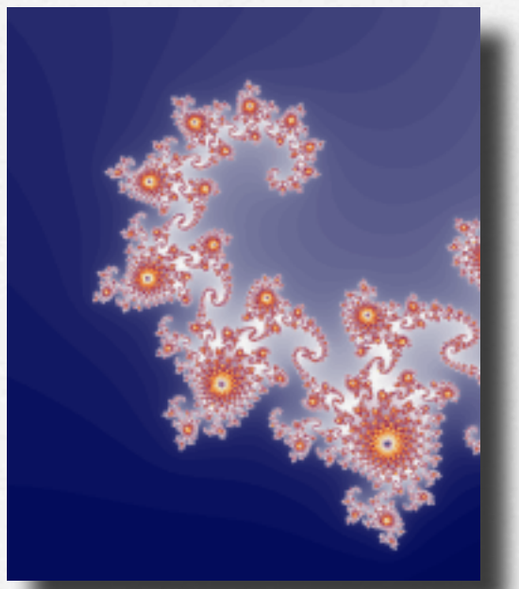


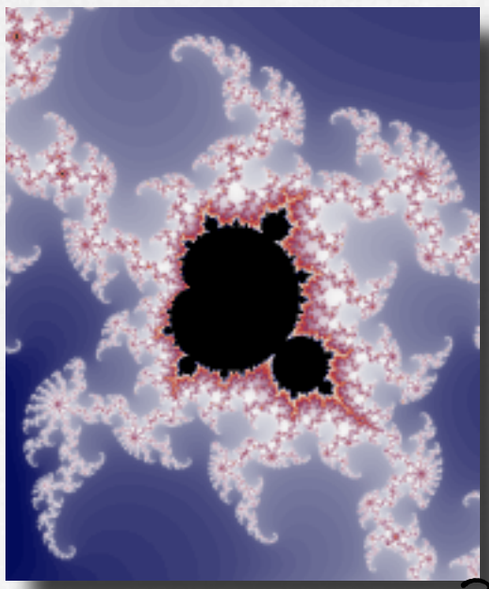


Roger Penrose

in "The Emperor's New mind", Oxford Univ. Press, 1994

A mathematical
example of emergent
structure





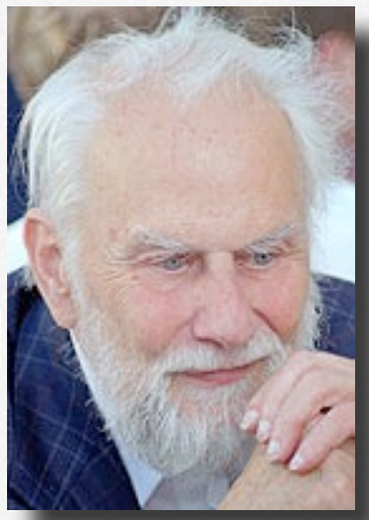
Emergence of patterns in Nature

1950s - Alan Turing proposes a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis, i.e., the development of form and shape in biological systems.

From website of the Biological Modeling and Visualization research group, Department of Computer Science at the University of Calgary:

See http://www.swintons.net/jonathan/turing.htm

Chaos, Order & Self-Organisation

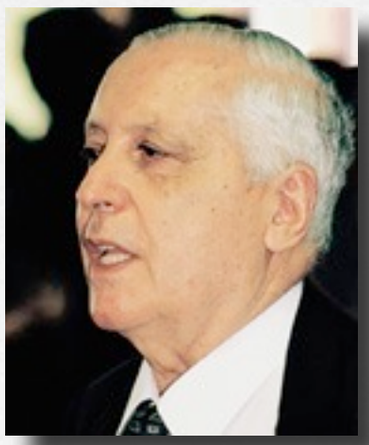


Hermann Haken

- SYNERGETICS - the study of the origins and evolution of macroscopic patterns and spacio-temporal structures in interactive systems

- Emphasis on mapping out self-organisational processes in science and the humanities - e.g. autopoiesis

- Mathematical modelling of nonlinear and irreversable processes, dissipative structures ...



Ilya Prigogine

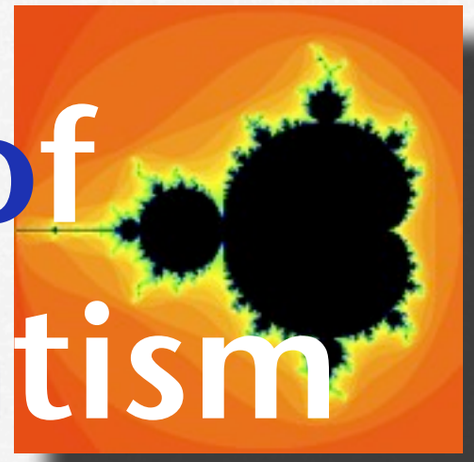
See: Michael Bushev, Synergetics - Chaos, Order, Self-Organization, World Scientific, 1994

Big Claims

Emergence is often invoked in an almost mystical sense regarding the capabilities of behavior-based systems. Emergent behavior implies a holistic capability where the sum is considerably greater than its parts. It is true that what occurs in a behavior-based system is often a surprise to the system's designer, but does the surprise come because of a shortcoming of the analysis of the constituent behavioral building blocks and their coordination, or because of something else?

Ronald C. Arkin in "Behaviour-Based Robotics", MIT Press, 1998, p.105

The Rise & Fall of British Emergentism



" ... the characteristic behaviour of the whole ... could not, even in theory, be deduced from the most complete knowledge of the behaviour of its components ... This ... is what I understand by the 'Theory of Emergence'. I cannot give a conclusive example of it, since it is a matter of controversy whether it actually applies to anything ... I will merely remark that, so far as I know at present, the characteristic behaviour of Common Salt cannot be deduced from the most complete knowledge of the properties of Sodium in isolation; or of Chlorine in isolation; or of other compounds of Sodium, ..."

- C.D. Broad, The Mind and Its Place In Nature, Kegan-Paul, London, 1925, p.59

C. D. Broad 1887-1971

A Test for Emergence

1) Design: The system has been constructed by the designer, by describing local elementary interactions between components (e.g., artificial creatures and elements of the environment) in a language $\mathfrak{L}_1$.

2) Observation: The observer is fully aware of the design, but describes global behaviors and properties of the running system, over a period of time, using a language $\mathfrak{L}_2$.

3) Surprise: The language of design $\mathfrak{L}_1$ and the language of observation $\mathfrak{L}_2$ are distinct, and the causal link between the elementary interactions programmed in $\mathfrak{L}_1$ and the behaviors observed in $\mathfrak{L}_2$ is non-obvious to the observer - who therefore experiences surprise. In other words, there is a cognitive dissonance between the observer's mental image of the system's design stated in $\mathfrak{L}_1$ and his contemporaneous observation of the system's behavior stated in $\mathfrak{L}_2$.

Ronald, Sipper and Capcarrère in "Design, observation, surprise! A test of emergence", Artificial Life, 5 (1999), 225-239

Descriptions and Emergent Structure

- Notice - It is often possible to get descriptions of emergent properties in terms of the elementary actions

- E.g., this is what Turing did for the role of Fibonacci numbers in relation to the sunflower etc.

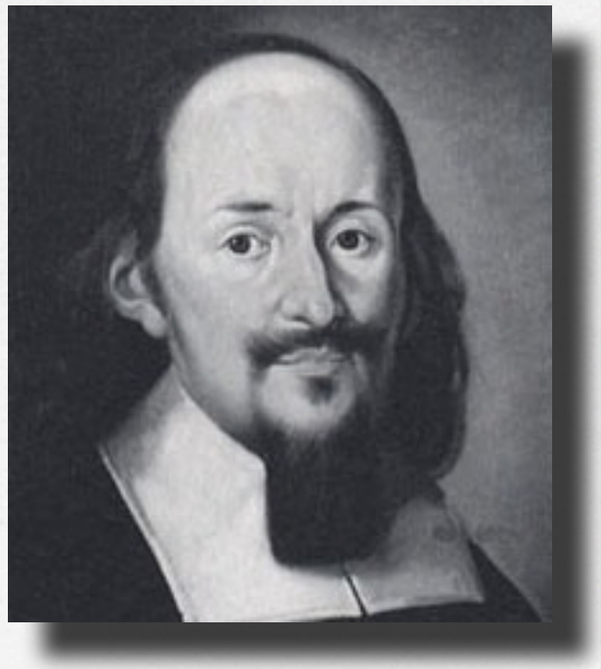
- In mathematics, it is well-known that complicated descriptions may take us beyond what is computable

- A potential source of surprise in emergence ...

Descriptions and Emergent Structure ...

- Intuition - entities exist because of, and according to, mathematical laws. In the words of Leibniz [1714] -

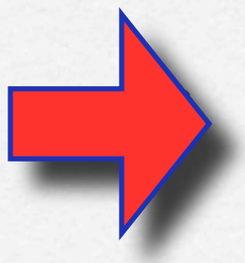
- 'The Monadology', section 32:
  " ... there can be found no fact that is true or existent, or any true proposition, without there being a sufficient reason for its being so and not otherwise, although we cannot know these reasons in most cases."



Definability the key concept

Alfred Tarski

- That is – natural phenomena not only generate descriptions, but arise and derive form from them . . .



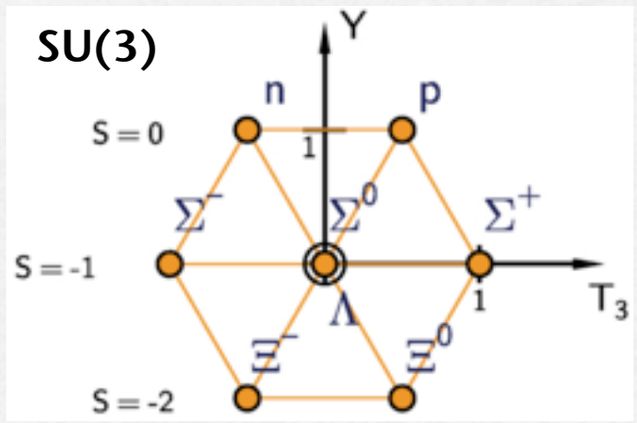
... so – connecting with a useful abstraction – that of mathematical definability – or, more generally, invariance (under the automorphisms of the appropriate structure) ...

- ... giving precision to our experience of emergence as a potentially non-algorithmic determinant of events

Definability and Symmetries

Murray Gell-Mann

- Symmetries play a huge role in science ..



- ... expressing appropriate automorphisms

- ... or particular lapses in definability

... so giving a clear route: from fundamental mathematical structures, and their automorphisms and breakdowns in definability - to far-reaching macro-symmetries in nature

I believe the
following aspects of evolution to be true, without knowing how to turn them into (respectable) research topics.

Important steps in evolution are robust. Multicellularity evolved at least ten times. There are several independent origins of eusociality. There were a number of lineages leading from primates to humans. If our ancestors had not evolved language, somebody else would have.

Martin Nowak,

Director, Program for Evolutionary Dynamics, Harvard University,

in John Brockman (ed.): "What We Believe But Cannot Prove"

3. The Challenge of Modelling Mentality

"At first Poincaré attacked [a problem] vainly for a fortnight, attempting to prove there could not be any such function ... [quoting Poincaré]:

'Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it ... I did not verify the idea ... I went on with a conversation already commenced, but I felt a perfect certainty.

On my return to Caen, for conscience sake, I verified the result at my leisure.' "

from Jacques Hadamard [1945], "The Psychology of Invention in the Mathematical Field", Princeton Univ. Press

More Emergence and Mathematical Intuition

Jacques Hadamard
1865-1963

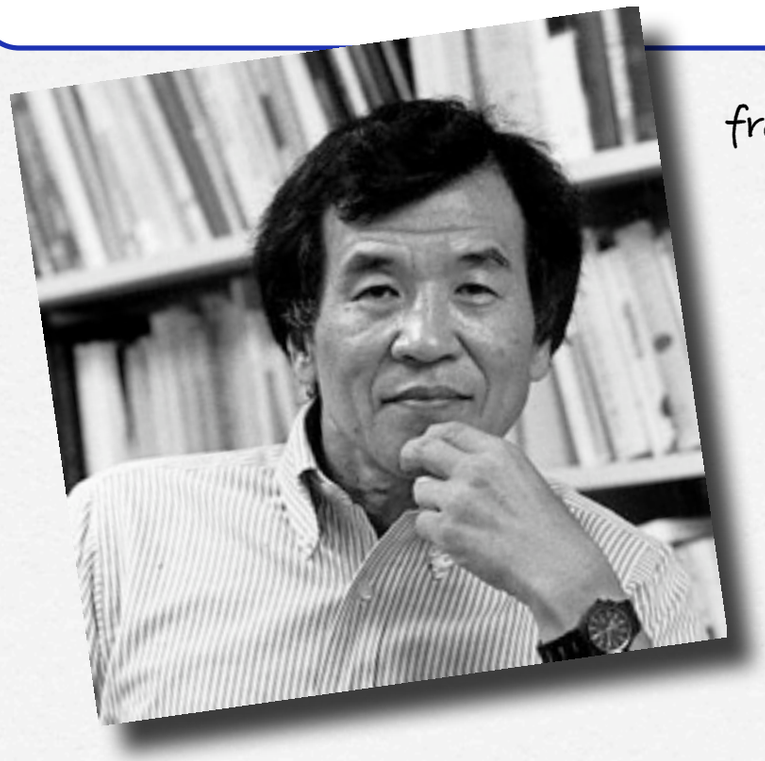
Jacques Hadamard again, again quoting Poincaré:

'Then I turned my attention to the study of some arithmetical questions apparently without much success ... Disgusted with my failure, I went to spend a few days at the seaside and thought of something else. One morning, walking on the bluff, the idea came to me, with just the same characteristics of brevity, suddenness and immediate certainty, that the arithmetic transformations of indefinite ternary quadratic forms were identical with those of non-Euclidian geometry.'

from Jacques Hadamard [1945], "The Psychology of Invention in the Mathematical Field", Princeton Univ. Press

Supervenience

Supervenience 'represents the idea that mentality is at bottom physically based, and that there is no free-floating mentality unanchored in the physical nature of objects and events in which it is manifested'



from Jaegwon Kim: "Mind in a Physical World", MIT Press, 1998, pp.14-15

"A set of properties A supervenes upon another set B just in case no two things can differ with respect to A-properties without also differing with respect to their B-properties."

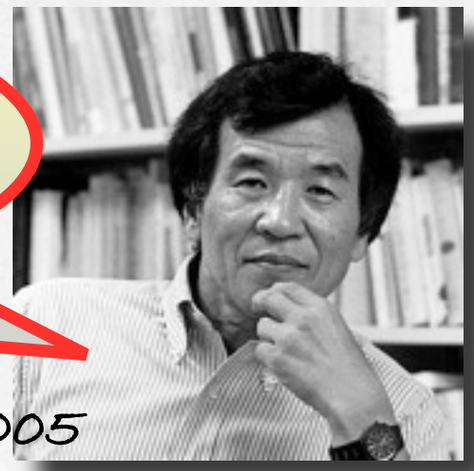
Stanford Encyclopedia of Philosophy

The Problem of Mental Causation

- How can mentality have a causal role in a world that is fundamentally physical?

- And what about 'overdetermination' - the problem of phenomena having both mental and physical causes?

... the problem of mental causation is solvable only if mentality is physically reducible; however, phenomenal consciousness resists physical reduction, putting its causal efficacy in peril.



-Jaegwon Kim: Physicalism, or Something Near Enough, Princeton, 2005

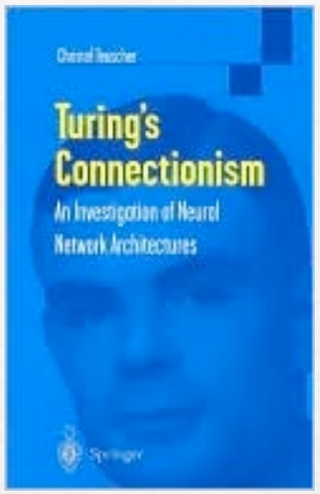
Connectionist Models to the Rescue?

There is a reasonable chance that connectionist models will lead to the development of new somewhat-general-purpose self-programming, massively parallel analog computers, and a new theory of analog parallel computation: they may possibly even challenge the strong construal of Church's Thesis as the claim that the class of well-defined computations is exhausted by those of Turing machines.

Paul Smolensky [1988] (recipient 2005 David E. Rumelhart Prize),

On the proper treatment of connectionism, in Behavioral and Brain Sciences, 11, pp. 1-74

Connectionist Models to the Rescue?



- These have come a long way since Turing's [1948] discussion of 'unorganised machines', and McCulloch and Pitts [1943] early paper on neural nets

- But for Steven Pinker "... neural networks alone cannot do the job".

- And focussing on our elusive higher functionality, he points to a "kind of mental fecundity called recursion" . . .

Emergent mental images recycled

We humans can
take an entire proposition and give it a role in some larger proposition. Then we can take the larger proposition and embed it in a still-larger one. Not only did the baby eat the slug, but the father saw the baby eat the slug, and I wonder whether the father saw the baby eat the slug, the father knows that I wonder whether he saw the baby eat the slug, and I can guess that the father knows that I wonder whether he saw the baby eat the slug, and so on.

Steven Pinker,

How the Mind Works, W. W. Norton, New York, 1997

Definability as a Key to Representation

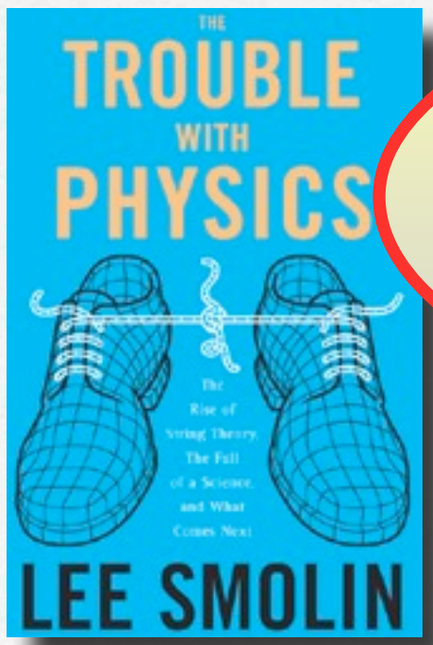
"As the brain forms images of an object - such as a face, a melody, a toothache, the memory of an event - and as the images of the object affect the state of the organism, yet another level of brain structure creates a swift nonverbal account of the events that are taking place in the varied brain regions activated as a consequence of the object-organism interaction. The mapping of the object-related consequences occurs in first-order neural maps representing the proto-self and object; the account of the causal relationship between object and organism can only be captured in second-order neural maps. ... one might say that the swift, second-order nonverbal account narrates a story: *that of the organism caught in the act of representing its own changing state as it goes about representing something else.*"

- Antonio Damasio [1999], The Feeling Of What Happens, p.170

Definability in What Structure?

- In modelling the physical universe –

… causality itself is fundamental



Lee Smolin, 'The Trouble With Physics', p.241

Definability in What Structure?

- Early champions of the role of causality - Roger Penrose, Rafael Sorkin, Fay Dowker, and Fotini Markopoulou

It is not only the case that the spacetime geometry determines what the causal relations are. This can be turned around: Causal relations can determine the spacetime geometry ...
It's easy to talk about space or spacetime emerging from something more fundamental, but those who have tried to develop the idea have found it difficult to realize in practice. ... We now believe they failed because they ignored the role that causality plays in spacetime. These days, many of us working on quantum gravity believe that causality itself is fundamental - and is thus meaningful even at a level where the notion of space has disappeared.

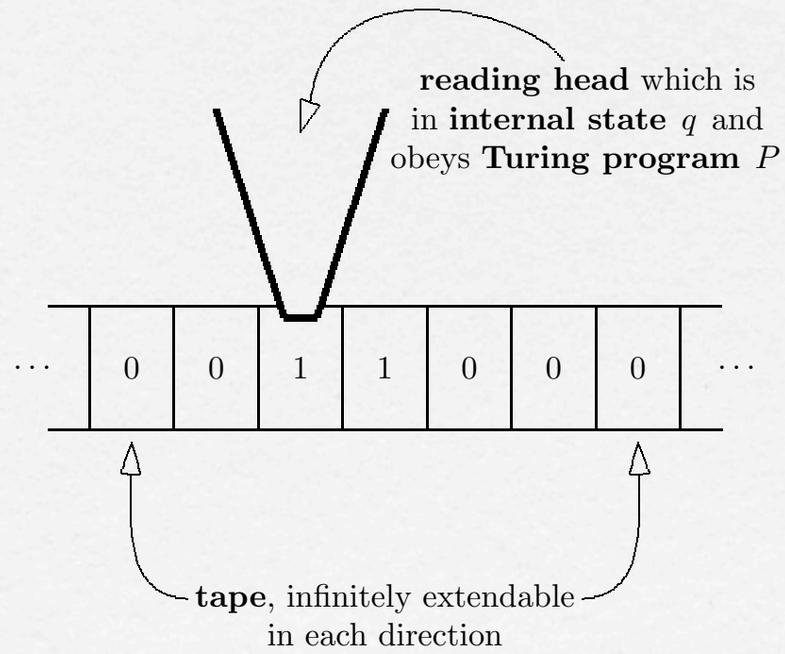
Lee Smolin, The Trouble With Physics, p.241

The Turing model extended ...

- 1939 - Turing's *oracle Turing machines*

- Provide a model of *computable content* of *structures*, *based on p.c. functionals over the reals*

- A model *within which Newtonian computability etc comfortably fit ...*



The Turing model extended ...

- 1939 - Turing's *oracle Turing machines*

- Provide a model of *computable content* of *structures*, *based on p.c. functionals over the reals*

- 1944 - Post defines the *degrees of unsolvability* as a *classification of reals* in terms of their relative computability

Phyllis, Emil and Gertrude Post

- Giving *a landscape with a rich structure*

… Using Real-time Auxiliary Data

- 1954 - Kleene's notion of Partial Recursive In
- 1959 - Friedberg and Rogers - Enumeration Reducibility
- 1961 - Myhill/Skordev - Relative computability of partial functions (extending Davis, Kleene)…

Rich research area - Including the Bulgarian School of D. Skordev, I. Soskov, A. Soskova, A. Ditchev, H. Ganchev, M. Soskova and others

See: http://www.fmi.uni-sofia.bg/fmi/logic/skordev/history.htm

The Turing landscape, causality and emergence ...

- can describe global relations in terms of local structure ...

- ... so capturing the emergence of large-scale formations

Mathematically - formalise as definability over structure based on Turing functionals?

More generally - as Invariance under automorphisms

Hartley Rogers' programme ...

Fundamental problem: Characterise the Turing invariant relations

- Intuition: These are key to pinning down how basic laws and entities emerge as mathematical constraints on causal structure

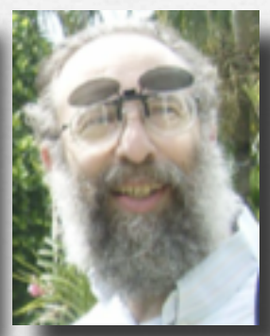
- Notice: The richness of Turing structure discovered so far becomes the raw material for a multitude of non-trivially definable relations

Bi-interpretability

Bi-interpretability Conjecture (Harrington): The Turing definable relations are exactly those with information content describable in second-order arithmetic

- Notice: Conjecture rules out there being non-trivial Turing automorphisms ...





- But: Work over the years makes it increasingly unlikely ...

An Informational Universe

Described *in terms of reals* … With natural laws based on *algorithmic relations between reals*

Emergence described in terms of *definability/invariance*

. . . with failures of definable information content modelling *mental phenomena*, *quantum ambiguity*

. . . which gives rise to *new levels of computable structure*

. . . and a *fragmented scientific enterprise*

Thank you!